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SPECIAL SECTION—ASSESSMENT OF SCHEMES FOR EARTHQUAKE PREDICTION

Decision-theoretic limits on earthquake prediction

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SUMMARY

I apply Decision Theory to the question of how accurate earthquake predictions must be to serve as a reliable basis for action. Even with optimistic estimates of the parameters involved, the lower bound on the required accuracy is extraordinarily demanding, being over 10 times higher than that of current meteorological forecasts. Given the abruptly self-organizing nature of earthquakes, it is extremely unlikely that precursors can attain such levels of accuracy. I therefore conclude that prediction of major earthquakes is, in any practical sense, impossible.

Key words: Decision Theory, earthquake prediction.

1 INTRODUCTION

The search for ways of predicting major earthquakes is one of the most enduring endeavours of modern geophysics, having continued intermittently for over a century. The ultimate aim of this endeavour is simply stated: to predict reliably and with reasonable precision both the timing and location of major earthquakes. The prototypical approach to this problem is to search for 'precursors' of major quakes that occur on timescales allowing civil authorities to initiate damage limitation. To date, however, not a single reliable precursor has been identified (Main 1996); nevertheless, the search goes on (Mogi 1994). The implicit assumption behind such doggedness is that reliable precursors do exist, and will eventually be found, given both the determination and the resources. This assumption has, however, been challenged for many years (see e.g. Macelwane 1946), principally on the grounds that unimpeachable evidence for the existence of reliable precursors has never emerged.

In what follows, I address a question of interest to both critics and advocates of earthquake prediction: what do we really want from a useful prediction method? More specifically, what are the characteristics any putative earthquake precursor must have to make the resulting prediction of practical value? Answers to this question usually involve somewhat vague concepts, like 'accuracy' and 'usefulness'. As I shall show, Decision Theory allows us to capture these concepts precisely, and to set reasonable bounds on them. Crucially, it also leads to a *minimum*-accuracy figure any precursor must achieve if it is to serve as a reliable basis for action. I shall show that, even using generous values for various key parameters, this lower bound is extremely demanding, essentially because of the infrequency with which major quakes strike. The results effectively rule out any hope of making practically useful predictions of major quakes.

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2 WHAT DO WE WANT FROM A PREDICTION METHOD?

The failure to date of all attempts to identify a reliable quake precursor does not *ipso facto* imply that such precursors do not exist. Indeed, five phenomena (pre-shocks, groundwater rise, seismic quiescence, foreshocks and radon concentrations) are currently on the IASPEI Preliminary List of Significant Earthquake Precursors (Wyss & Booth 1997). With due deference to those who insist that past failure strongly suggests more of the same, I will henceforth assume that the search for precursor phenomena is not completely hopeless, and pose the following question: how reliable do they have to be to give civil authorities confidence in taking reasonable mitigating action?

Mathematically, this question can be recast as follows: under what conditions are we justified in taking a decision, D, based on a prediction of a quake event Q? These conditions can be found using the probabilistic framework of Decision Theory (e.g. Lindley 1985). To begin the analysis, we note that the quake can either occur (Q) or not occur ($\sim Q$), the prediction being of a quake striking (S) or not ($\sim S$), and that we can either decide to take the prediction seriously and base a decision upon it (D), or ignore it ($\sim D$). There are four outcomes of the various decisions, each characterized by a joint probability: for example $\Pr(Q \ \& \ \sim D)$ represents the probability of the quake occurring and our deciding to ignore a prediction of it. Each of these four outcomes will also have a consequence, characterized by a pure number L quantifying the loss resulting from a specific combination of event and decision. We can then form a total loss resulting from the various outcomes:

$$L_{tot} = L_{00}\Pr(Q\ \&\ D) + L_{11}\Pr(\sim Q\ \&\ \sim D) + L_{10}\Pr(\sim Q\ \&\ D) + L_{01}\Pr(Q\ \&\ \sim D). \quad (1)$$

To incorporate the explicit effect of our response to the prediction S on the total loss, expand each of the joint probabilities in (1):

$$\Pr(Q\ \&\ D) = \Pr(Q\ \&\ D|S)\Pr(S) + \Pr(Q\ \&\ D|\sim S)\Pr(\sim S), \text{ etc.}, \quad (2)$$

where the vertical bar implies 'given'. By Bayes' Theorem,

$$\Pr(Q\ \&\ D|S) = \Pr(Q|S\ \&\ D)\Pr(D|S) = \Pr(Q|S)\Pr(D|S), \text{ etc.}, \quad (3)$$

where I have used the fact that the occurrence of Q is independent of D. These four probabilities inserted into (1) lead to

$$L_{tot} = m[B\Pr(\sim Q) - A\Pr(Q)K] + n[(1-B)\Pr(\sim Q) - (1-A)\Pr(Q)K]. \quad (4)$$

Here $m = \Pr(D|S)$ and $n = \Pr(D|\sim S)$ summarize our response to the prediction: if we always put our trust in its prognostication, then $m=1$, $n=0$, while always ignoring it corresponds to $m=0$, $n=0$. The accuracy of the prediction method is reflected by $A = \Pr(S|Q)$ and $B = \Pr(S|\sim Q)$; their ratio, $A/B = \Pr(S|Q)/\Pr(S|\sim Q)$ is the so-called likelihood ratio (LR) of the prediction method, and is conventionally used to capture the accuracy of the prediction method; $LR > 1$ constitutes some predictive ability. Finally, K is the so-called loss structure:

$$K = (L_{01} - L_{00})/(L_{10} - L_{11}), \quad (5)$$

which measures the relative losses due to complacency—that is failing to take action and the quake striking—and overreaction, with action being taken and the quake not striking.

Our criterion for deciding when to take a quake prediction seriously can now be stated mathematically: we do so whenever the total loss caused by putting our faith in the prediction is less than the loss caused by ignoring it. The former is equivalent to setting $m=1$, $n=0$, while the latter corresponds to $m=n=0$. From (4), one then finds that for a prediction of a major quake to serve as a justifiable basis for decisions, it must satisfy the inequality

$$LR\ \text{Odds}(Q)K > 1, \quad (6)$$

where $\text{Odds}(Q) = \Pr(Q)/\Pr(\sim Q)$, the 'base rate' for major quakes.

It should be stressed that the criterion (6) is a *lower* bound, that is the *minimum* accuracy rate any putative precursor of major quakes must achieve to make its predictions worth taking seriously. Before we can calculate its numerical value, however, we must tackle the question of setting the various quantities it contains.

3 SETTING THE BASE RATE, ODDS(Q)

To apply the criterion (6), we must first be clear about what each of the terms means. The base rate for major quakes is the odds on a major quake occurring within the time-frame of the precursor. For example, if a precursor is claimed to give warnings of quakes with a precision of ≈ 1 week, the appropriate base rate is the odds of a major quake occurring within an interval of ≈ 1 week. As intuition suggests, and (6) confirms, the higher the base rate, the less accurate the precursor needs to be: that is broad-brush forecasts of frequent events are less demanding than precise predictions of rare ones. To calculate the appropriate base rate for quakes of magnitude M on a timescale τ we use (Lomnitz 1994)

$$\text{Odds}(M, \tau) = \exp[\alpha\tau \exp(-\beta M)] - 1, \quad (7)$$

where α is the mean number of earthquakes ($M > 0$) over the interval τ, and β is the so-called reciprocal mean magnitude for the region in question. For California, seismic records show that $\alpha \approx 10^5$ per annum while $\beta \approx 2.0$. To calculate the characteristic base rate for a major quake, let us set $M = 7.8$; this is equivalent to the San Francisco quake of 1906, and it is also the magnitude of the event at the heart of Japan's Earthquake Countermeasures Law, which has motivated much research into earthquake prediction. We then find from (7) the base rates over various timescales shown in Table 1.

4 SETTING THE LOSS STRUCTURE, K

As the definition of (5) involves no fewer than four different losses, setting K appears far from simple. For example, one might piously argue that the loss of one single life is worth any number of false alarms, making K huge. The economic costs of false alarms are, however, hardly insignificant (one estimate puts the cost at around \$7 billion a day for Tokyo; Geller 1997), and even states of heightened alert are impossible to maintain indefinitely. Gauging the impact of all this on K is far from simple; fortunately, however, there is a way of estimating K from a more manageable parameter, namely the lowest odds on a prediction proving correct for which one feels justified in taking action, $\text{Odds}(Q|S)_{\min}$. It can be shown that (Matthews 1997)

$$K = 1/\text{Odds}(Q|S)_{\min}. \quad (8)$$

This reflects the intuitive logic that the higher the potential loss from complacency compared to overreaction—and thus the higher K becomes—the lower the odds at which taking a prediction seriously seems justified. Setting K can now be done by drawing on real-life experience of setting $\text{Odds}(Q|S)_{\min}$. The UK Meteorological Office issues Severe Weather Warnings—advising people to take simple measures such as avoiding unnecessary journeys—whenever the posterior odds of the severe weather exceed around 0.4 (UK Meteorological Office, personal communication, 1997); the US National Hurricane Center adopts a figure of ≈ 0.3 for issuing its 24 hr alert warnings (Olshansky, personal communication, 1997). It would

Table 1. Base-rate odds for a major ($M \approx 7.8$) quake over various time intervals.

Time intervals	1 hour	1 day	1 week	1 month	1 year	1 decade
Base-rate odds	2×10^{-6}	5×10^{-5}	3×10^{-4}	1×10^{-3}	2×10^{-2}	2×10^{-1}

be difficult to argue that any civic authority would feel justified in calling for the far more disruptive action of total evacuation of a city if posterior odds on a quake striking were lower than this figure. Thus, from (8), it seems reasonable to set $K \approx 3$ as an appropriate, if somewhat generous, loss structure for evacuation following a quake prediction.

Evacuation is, however, the most drastic action than can be based on a quake prediction. Many other ameliorative measures, such as the isolation of gas supplies and the preparation of blood banks, might be deemed appropriate at a much lower-risk level of concern. Clearly, such 'alert' measures are justifiable at much lower posterior probabilities—and thus higher K. Noting that such actions are deemed justifiable for hurricanes on posterior probabilities of ≈ 0.1, I set $K \approx 10$ for such measures.

I now consider the two types of precursor-based decision in turn, and examine their implications for the minimum level of accuracy demanded of the precursor.

5 PRECURSOR ACCURACY FOR EVACUATION

To calculate the minimum accuracy needed to justify evacuation decisions, one first needs the appropriate base rate, and thus timescale, of the prediction. This is necessarily a balance between giving the civil authorities sufficient time to organize an orderly evacuation, and minimizing the losses incurred from economic 'down-time' and social disruption. Only precursors capable of narrowing down the timing of a major quake to within ≈ 1 week could be considered for such a critical role; Table 1 then shows that the corresponding base rate is 3×10^{-4}. Setting $K \approx 3$ as discussed above, (6) then leads to

$$LR(\text{evacuation}) > 1000. \tag{9}$$

This is an extraordinarily demanding requirement; to put it in context, it is 200 times higher than the LR of ≈ 5 achieved by the UK Meteorological Office for its 24 hr predictions of rain (Matthews 1996). Yet, as already noted, not a single reliable earthquake precursor with an LR significantly greater than ≈ 1 has ever been identified. Furthermore, there is growing evidence that earthquakes are 'critical' phenomena which respond abruptly and catastrophically to small perturbations (Main 1996). As such, it seems highly unlikely that there exist quake precursors capable of giving predictions as accurate as those currently achieved by meteorologists, let alone 200 times more so.

Only huge increases in base rate and/or loss structure are capable of bringing the required LR down to a reasonable (i.e. single-figure) level; for example by requiring only very broad-brush predictions of major quakes on timescales of decades. However, both such measures serve only to undermine the original aim of the prediction enterprise: to give more than just broad-brush forecasts, and provide justifiable decisions on tight time-frames. The conclusion thus appears ineluctable—predictions of major ($M \approx 7$–8) quakes reliable enough to justify evacuation are impossible: such quakes are simply too infrequent to predict with the required reliability.

6 PRECURSOR ACCURACY FOR ALERTS

Alerts are considerably less demanding, and the ability to make them would be motivation for continuing with the search for precursors. To see what accuracy is required, we again require an appropriate timescale over which an alert could be sustained, and for which the precursor is deemed to be accurate. Watching alerts are called for hurricanes on timescales of ≈ 36 hr; once again adopting a generous figure for the quake prediction enterprise, let us set a timescale of ≈ 1 month. Setting $K \approx 10$ as discussed above, and using the values from Table 1, eq. (6) then leads to a *minimum* required level of accuracy of

$$LR(\text{alertness}) > 100. \tag{10}$$

While an order of magnitude less demanding than the figure for evacuation, this figure is still far higher than even the best meteorological forecasts can currently achieve—despite being based on generous assumptions. I therefore conclude that for major ($M \approx 7$–8) quakes, even predictions capable of justifying alert calls are not possible.

Given these two negative findings for major quakes, the question arises as to what magnitude of quake might be amenable to useful prediction. With the values of K and timescales used above, we can use (6) and (8) to extract the appropriate value of M needed to produce likelihood ratios comparable to those now achieved in meteorology. We find that evacuation decisions are only justifiable for $M \leq 5$, while alert calls are justifiable only for $M \leq 6$; the reliability of the precursor is undermined by the infrequency of quakes more severe than this. In view of the relatively minor effects of such quakes, and the unreliability of the same methods for any more serious quake, the huge effort currently made to find precursors becomes very hard to justify.

7 CONCLUSIONS

In this paper, I have used Decision Theory to answer a key question surrounding the use of putative precursors to predict earthquakes: how reliable must such a precursor be to form the basis of useful earthquake predictions? The answer relies on two key factors: the base rate of major quakes on the timescale of the precursor, and the relative costs of ignoring correct predictions and responding to false alarms. Even using generous values for these factors, the resulting *minimum* accuracy figure—as captured by the likelihood ratio—required to make useful predictions of major quakes is far in excess of that achieved by state-of-the-art meteorology. Given the persistent failure of any putative technique to show an LR exceeding unity, let alone the LRs achieved by meteorology, my conclusion is a blunt one: attempts to find methods of predicting major earthquakes are misguided. The above analysis supports instead the notion of broad-brush 'forecasts' of major quakes on a timescale of decades, which falls into the province of hazard mitigation. This appears to be a more fruitful approach to dealing with the global earthquake threat.

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